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Energy transfer in resonant and near-resonant internal wave triads in a bounded domain with varying topography

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Abstract

In oceans, internal gravity waves are often produced when the stably stratified ocean water is driven back and forth over submarine topography by tidal currents. Internal gravity waves can move through varying water depth in the ocean for very large distances (O(1000)km). Understanding the cascade of energy from the large scale internal waves to small scales helps in predicting/quantifying deep ocean mixing. Weakly nonlinear wave-wave interactions is one of the mechanisms through which the cascade may occur. In this paper, we derive a mathematical model for calculating the energy exchange in resonant and near-resonant triads consisting of weakly nonlinear internal gravity wave packets in a bounded domain with varying topography. We assume each internal wave in the triad to have a slowly varying amplitude and a rapidly varying phase (both in space and time), and derive the amplitude evolution equations using the method of multiple scales. In the presence of uniform stratification, when resonant triads interact in medium of varying fluid depth (h), the horizontal wavenumber condition, given by $k_{(1,a)}+k_{(2,b)}+k_{(3,c)}=0$ is unaffected. Here (a,b,c) are integers which denote what mode is the horizontal wavenumber in the vertically bounded domain. Also in the presence of uniform stratification, for any triad, the constituent waves' horizontal group speed is proportional to h, while the non-linear coupling coefficients in proportional to $1/h^2$. In the presence of non-uniform stratification, when three waves interact in a region of varying h, the horizontal wavenumber condition $k_{(1,a)}+k_{(2,b)}+k_{(3,c)}=0$ will be satisfied (or nearly satisfied) only when $a=b=c$. However, such triads are only possibly when $\omega \ll N$. Triads which do not satisfy the condition $a=b=c$ may not satisfy the horizontal wavenumber condition as h varies. Moreover, this is also true for self interaction of a mode. In the presence of non-uniform stratification, the non-linear coupling coefficients do not decrease (increase) monotonically with increasing (decreasing) domain height. The non-linear coupling coefficients may also change several orders of magnitude with change in h for both self interaction of a mode or a mode interacting with two other different modes. It is shown that, three waves interacting amongst themselves and propagating in a region of non-flat bottom topography may have reduced energy transfer due to a change of relative phase in between the waves of the triad. In general, this reduction of energy transfer is found to be more pronounced with increase in $(\partial h/\partial x)$.