Investigating the effectiveness of Padé-type approximations

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Abstract

Both algebraic and numerical Padé-type approximants represent improvements on more common approaches as far as accuracy is concerned. However, the derivation of an algebraic Padé approximation first requires the computation of a power series approximation to the target function, and Padé-type stencils typically involve more operations than other schemes. We attempt to quantify the improvement in accuracy expected alongside the associated increase in computational expense. We do so for a set of problems chosen to depict differing behaviours in a range of target functions. We apply the compact schemes introduced by Lele (which have since been used in problems requiring their "spectral-like" resolution) to various PDEs intended to represent a range of phenomena in fluid dynamics (e.g. the 2D Helmholtz equation, as found in fluid potential fields; the 1D, linear heat equation; and, the conservative form of the Burgers' equation). Of particular interest is the Van der Pol oscillator model which serves as an analogy for the near-wake flow oscillations observed in the von Karman vortex street of a slender bluff body in cross-flow. Measuring the accuracy and computational run-time of each method, we found that, for example, for the heat equation, we found a 1.8% average error reduction with a 51% increase in run-time for the 4th-order accurate Classical Padé-type stencil in comparison to the standard central difference stencil. Similar comparisons can be made for the algebraic approximation, although the run-time is reduced while the error improvement is enhanced (by comparison with a corresponding Taylor series approximation).